Issues on the Application of Wavelet to Construction Problems

Wavelet Application Areas

Wavelet application types

- Detecting discontinuity (incident detection)
- Denoising
- Feature extraction
- Detecting self-similarity
- Detecting long-term trend
- Pattern prediction
- Data compression

Wavelet Types

- CWT, DWT, and WPT
- Haar, redundant Haar
- Daubechies
- Symlets
- Coiflets
- Biorthogonal
- …

Which Wavelet?

- Freedom to choose a wavelet
  - Blessing or Curse?
- How much efforts need to be made for finding a good wavelet?
  - Any wavelet will do?
- What properties of wavelets need to be considered?

Regularity

- The order of regularity of a wavelet is the number of its continuous derivatives.
- Regularity can be expanded into real numbers.

If $\psi^{(m)}(t)$ resembles $|t - t_o|^r$ locally around t_o

then the regularity is $m + r$ with $0 < r < 1$.

- Regularity indicates how smooth a wavelet is.

Vanishing Moment

- Moment: *j's* moment of the function $\psi(t)$

$$\int_{-\infty}^{+\infty} t^{j}\psi(t)\,dt$$

- When the wavelet's k+1 moments are zero

i.e. $$\int_{-\infty}^{+\infty} t^{j}\psi(t)\,dt = 0 \quad \text{for} \quad j = 0,\ldots,k$$

the number of Vanishing Moment of the wavelet is k.

- Weakly linked to the number of oscillations.

Vanishing Moment (Cont'd)

- When a wavelet has k vanishing moments, suppression of signals that are polynomials of a degree lower than or equal to k is ensured.
- If a wavelet is k times differentiable, the wavelet has at least k vanishing moments.

Size of Support

- The number of FIR filter coefficients.
- The number of vanishing moments is proportional to the size of support.
- Trade-off between computational power required and analysis accuracy.
- Trade-off between time resolution and frequency resolution.

Comparisons

Db1 (Haar)	Db2 (D4)	Db5 (D10)	Db10 (D20)
R=0	R=0.5	R=1.59	R=2.90
VM=1	VM=2	VM=5	VM=10
SS=2	SS=4	SS=10	SS=20

The figures are from Mathworks (2010)

1D Signal (Cash Flow, Stock Index, Traffic Flow, Structural Vibration, Electrical …)

Extracting features and detecting self-similarity

- Choose the wavelet for your features.
- CWT could be effective for self-similarity detection.
- Our choice of wavelet?

The figure is from Mathworks (2010)

1D Signal (Cash Flow, Stock Index, Traffic Flow, Structural Vibration, Electrical …)

Denoising and detecting long-term trend

- Choose the wavelet that can produce better sparsity.
- Sufficient vanishing moments are required.
- Tradeoff between high vanishing moment and computational efficiency.
- Our choice of wavelet?

The figure is from Mathworks (2010)

1D Signal (Cash Flow, Stock Index, Traffic Flow, Structural Vibration, Electrical …)

Detecting discontinuity

- On different order of derivatives.
- Tradeoff between localized info. and high regularity.
- Our choice of wavelet?

The figure is from Mathworks (2010)

1D Signal (Cash Flow, Stock Index, Traffic Flow, Structural Vibration, Electrical …)

Data compression

- Choose the wavelet that can produce good sparsity.
- Tradeoff between the wavelet's resemblance to the signal and energy preserving capacity?
- Biorthogonal wavelets for boundary handling and image compression.
- The choice of our wavelet?

The figure is from Mathworks (2010)

Number of Decomposition Levels

- Number of decomposition levels can be determined from considering fractal.
- The choice of wavelet basis affect the possible number of decomposition levels.
- Raw data resolution is quite important.
- After all, usefulness determines the level of decomposition.

Conclusions & Recommendations

1. No clear-cut answer is available as to how to choose the optimum wavelet.
2. Each specific analytical aspect of each particular application deserves a review of theoretical wavelet properties.
3. Preliminary wavelet analyses on the signal to be analyzed can be very helpful in choosing the right kind of wavelet basis.

References

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